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SIMPLIFIED ANALYSIS OF SPLIT RING RESONATOR USED IN BACKWARD META-MATERIAL

Silvio Hrabar, Juraj Bartolic

Department of Radiocommunications and Microwave Engineering, University of Zagreb,
Unska 17, Zagreb, HR-10000, Croatia, e-mail: Silvio.Hrabar@fer.hr

ABSTRACT

A simple equivalent circuit of recently introduced split ring resonator (SRR) is proposed in this paper. It is shown that in the vicinity of resonant frequency, the SRR can be thought of electrically small, capacitively loaded loop antenna. Due to resonant behaviour of the antenna current, intensity of the magnetic field of the incoming plane wave may be locally decreased yielding a stop-band with negative effective permeability. Theoretical analysis was verified by measurements of the transmission coefficient of experimental structures in rectangular waveguide, in 10 GHz frequency band.

INTRODUCTION

Recent introduction of ‘left-handed’ (or ‘backward’) meta-material (material with both $\epsilon < 0$ and $\mu < 0$) [1] has attracted a lot of attention. In such a material, the wave vector \mathbf{k} and Poynting vector \mathbf{P} are anti-parallel, causing reversal of some basic electromagnetic phenomena such as Snell law and Doppler effect. In original design [1], the negative permittivity was achieved with an array of thin wires, for which is well known to have dielectric function similar to that from dilute plasma. Negative permeability was achieved by a new type of inclusion coined ‘split ring resonator’, (SRR). So far, properties of SRR have been analysed numerically [1] and literature is sparse of simple engineering model.

ANALYSIS OF SPLIT RING RESONATOR

The split ring resonator used in design of the first backward meta-material [1] is sketched in Fig.1.

The SRR comprises two conductive rings printed on a thin dielectric substrate and separated by

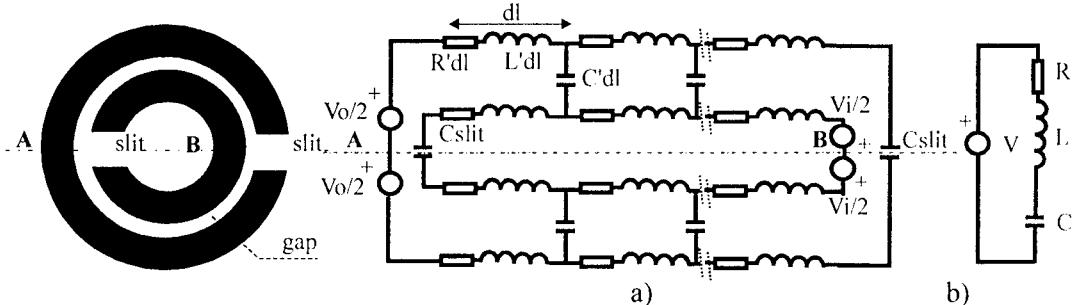


Fig. 1 The split ring resonator (SRR)

Fig. 2 a) The complete equivalent circuit of the SRR
b) The simplified approximate equivalent circuit

a narrow gap. Each ring has a slit, and rings are oriented in such a way that slits are on the opposite sides of line of symmetry. The electrical dimensions of the SRR are much smaller than a wavelength of the impinging plane wave. Bearing this fact in mind, one can think of the SRR as two mutually coupled small loop antennas. The voltage induced in each antenna can be simply calculated from Faraday law:

$$V = -j\omega \mu_0 A H_i \cdot \quad (1)$$

Here, V stands for induced voltage, ω stands for radial frequency, A and μ_0 stand for the loop area and free-space permeability, respectively. The symbol H_i stands for incident magnetic field,

which is assumed being perpendicular to the loop. The induced voltages are modelled as simple voltages sources located at point A (the source V_0 - outer ring) and point B (the source V_i - inner ring). The complete equivalent circuit is sketched in Fig. 2a. The symbols R' and L' stand for distributed resistance and inductance respectively, C_{slit} stands for capacitance of the slits and C' stands for distributed capacitance of the gap between rings. In circuit from Fig. 2a one identifies two main loops (inner and outer ring) which are connected across the gap via distributed capacitance C' . It is important to notice that voltage V_0 is always higher than voltage V_i by the ratio of areas formed by outer and inner ring, respectively (square of the ratio of radii of outer and inner rings). In SRR developed in [1], voltage V_0 is approximately two times higher than voltage V_i . Thus, current essentially flows from the outer ring into inner ring across the gap. It flows through many branches formed by distributed capacitance C' . Due to this branching, current in outer ring changes with the location at the ring. It is maximal at point A, then decreases along the ring and reaches minimal value at the slit. All the currents, which flow from the outer ring into the inner ring, of course, contribute to the net current in inner ring. Therefore, the current in inner ring exhibits maximum at the location of voltage source V_i (point B), then decreases along the ring and reaches minimum at the slit. Contribution of the current which flows across the slit (through the C_{slit}) to the net ring current is negligible, thus one can actually consider that $C_{slit} = 0$. Now, taking into account fact that currents flow predominantly across the gap, one can approximate the whole circuit with a much simpler circuit sketched in Fig. 2b. It comprises a single voltage source and a serial tank circuit. It actually means that a single, electrically small loop antenna loaded with a capacitor should behave very similarly to the SRR.

The simple analysis published elsewhere [3] shows that the magnetic field across the capacitively loaded loop antenna illuminated by a plane wave exhibits resonant behaviour:

$$H = H_i \left(1 - j \frac{K \omega \mu_0 A}{R + j \omega L - j / \omega C} \right) . \quad (2)$$

Here, R and L stand for intrinsic resistance and inductance of the loop, respectively, K is a constant, which describes geometry, and C denotes capacitance of the load. Alteration of the local magnetic field by the scattered field given by (2) is behaviour analogues to the magnetization of the magnetic material. Calculated intensity of the resultant local magnetic field (normalised on the incident field) is sketched in Fig. 3.

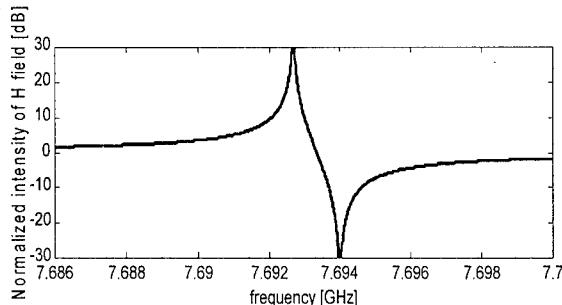


Fig. 3 Calculated local magnetic field

Loop parameters: $K=1\text{m}^{-1}$, $R=0.1\ \Omega$,
 $L=26\text{ nH}$, $C=0.016\text{ pF}$, $S=0.07\text{ cm}^2$

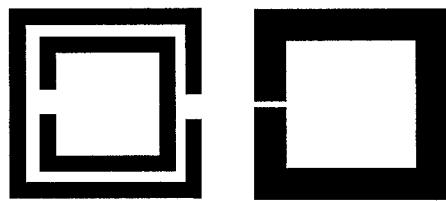


Fig. 4 Left Experimental SRR

Right Experimental capacitively loaded loop

It can be seen that intensity of the resultant local magnetic field can be either higher or lower than intensity of the incident field. Thus, it is possible to achieve both paramagnetic ($\mu>0$) and diamagnetic ($\mu<0$) behaviour. Please note that curve in Fig. 3 is analogues to the curve of relative permeability of the SRR published in [2].

EXPERIMENT

Two experimental structures have been fabricated on the ComClad substrate (thickness 0.5 mm, $\epsilon_r=2.6$). The first structure (Fig. 4 - left) is the SRR, dimensions of which were scaled from those in [1] in order to achieve resonance around 9 GHz. The second structure (Fig. 4 - right) is a single loop loaded with a capacitor (narrow gap). The capacitance and inductance were calculated using approximate equations from the standard handbook [4]. Each structure was inserted into standard rectangular X-band waveguide. The structures were always oriented in such a way that there is a component of the magnetic field perpendicular to the loop (or rings), i.e., the substrate was perpendicular to the waveguide walls. The waveguide was excited in dominant TE 01 mode and S_{21} parameter was measured by HP 8720B network analyzer. The measurement results are given in Fig.5 and Fig.6.

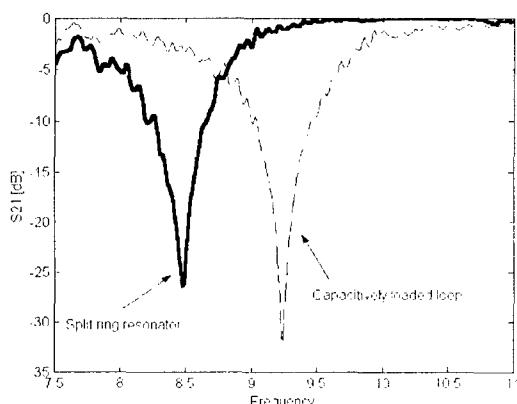


Fig 5. Magnitude of measured S_{21} parameter

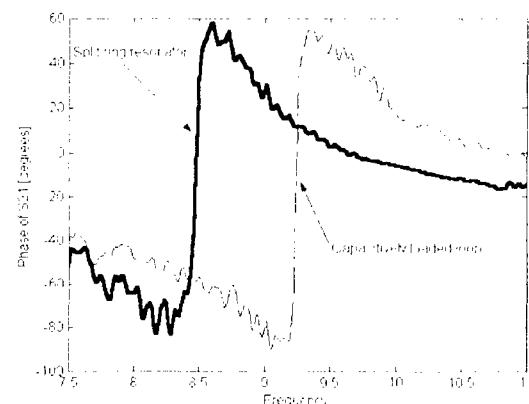


Fig 6. Phase of measured S_{21} parameter

One can notice that resonant frequencies of SRR and loaded loop are different, which is consequence of approximate equations used for design of the loop. Apart from this, behaviour of these two structures are very similar. Both structures exhibit a notch in transmission characteristic (Fig. 5) which corresponds with the serial resonance. Also, the curves of phase of S_{21} parameter (Fig. 6) are very similar. It proves that SRR can indeed be thought of a small capacitive loaded loop antenna and background physics is essentially the same. By extension of above analysis, the approximate equation for effective permeability of an array of capacitively loaded loop antennas was derived. Also, the appropriate negative permeability meta-material was designed, fabricated and successfully tested. Details of this research can be found elsewhere [3].

CONCLUSIONS

It is shown that SRR can be thought of capacitively loaded loop antenna. Such an antenna may increase or decrease local intensity of magnetic field of the incoming plane wave yielding both paramagnetic ($\mu>0$) and diamagnetic ($\mu<0$) behaviour.

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